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TWENTY-FIRST PROGRESS REPORT

on

CALIBRATION AND EVALUATION OF SKYLAB ALTIMETRY FOR GEODETIC DETERMINATION OF THE GEOID (Contract NAS9-13276, EPN 440) March 1, 1975 to March 31, 1975

to

NASA JOHNSON SPACE CENTER
Principal Investigation Management Office
Houston, Texas 77058

April 14, 1975

A. G. Mourad, Principal Investigator
S. Gopalapillai, M. Kuhner and D. M. Fubara, Co-Investigators

Z. H. Byrns, Code TF6 - NASA/JSC Technical Monitor

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PROGRESS

During this reporting period, we

(1) Completed the mathematical development and computer program for the collocation technique to filter the altimetry data. The mathematical development of this technique is presented in Appendix A.

(2) Continued studies of the correlation of the geoid profiles with the data from existing bathymetric and gravity maps. The long awaited bathymetric maps have been received from the Department of Defense. We also received 41 average free air gravity anomaly data values from NASA/GSFC. Most of these data correspond to EREP passes #4, #6, and #7. Pass #9 has only one value.

(3) Received several documents from NASA/JSC (see Appendix B).

DATA PROCESSING RESULTS

The Collocation technique described in Appendix A has been applied to the EREP pass #7 data. These data are selected owing to their very low signal to noise ratio. The numerical covariance function for the geoid height anomalies described by Tcherning and Rapp (see references at the end of Appendix A) was used to compute the covariance matrix, C_{ss}, described in equation (A-4). The error covariance matrix C_{nn} was assumed to be diagonal as follows

$$C_{nn} = \sigma^2 I$$

where I is an identity matrix and σ^2 is the error variance of the altimeter observations. Since the degree of smoothing depends on σ, two values (1 m and 2 m) were assumed for σ. Furthermore, the design matrix described in equation (A-1) is also set to zero.

The filtered data (signal, s) are, then, adjusted with the ground truth geoid heights to recover the biases, if any, in the segments of the data corresponding to the various submodes of the altimeter.

The profiles corresponding to both the filtered and unfiltered data minus the recovered biases are shown on Figure 1. The precision of the altimetry data assumed for this analysis is one meter. The profile for the filtered data is shown by a thick line, and that corresponding to the unfiltered data is indicated by a thin line. The profile of the ground truth data is shown on the same figure by a broken line. The corresponding results for the altimeter precision of two meters is presented on Figure 2. These figures indicate the viability of this technique in obtaining a set of realistic geoidal information from a set of noisy altimetry data. They also show the varying degree of smoothing depending on the assumed precision of the altimetry data.

PROBLEMS

There are no problems worth reporting during this period.

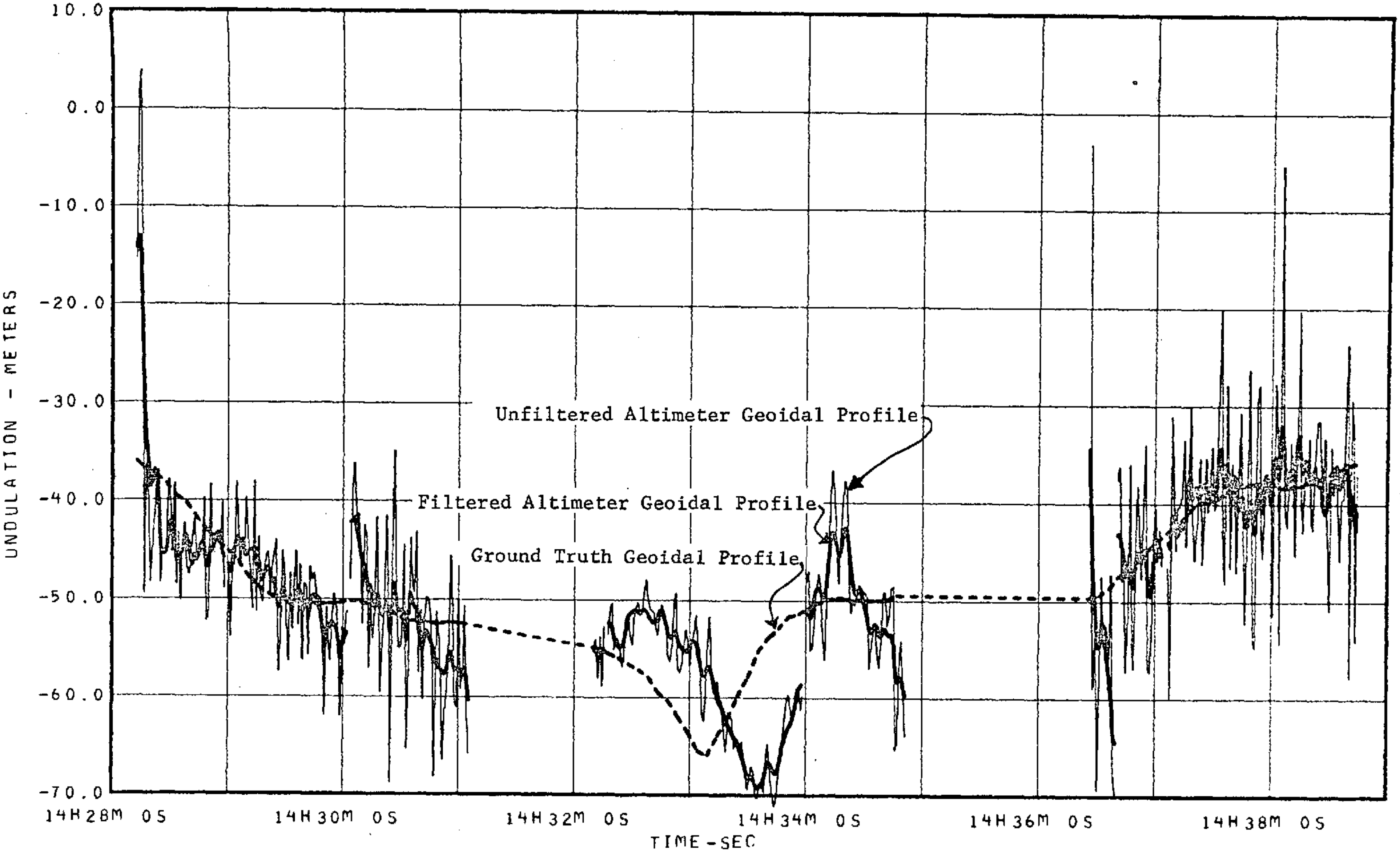

FIGURE 1. GEOID UNDULATIONS COMPUTED FROM SKYLAB PASS-7 ALTIMETRY DATA
ALTIMETER PRECISION = 1 m.

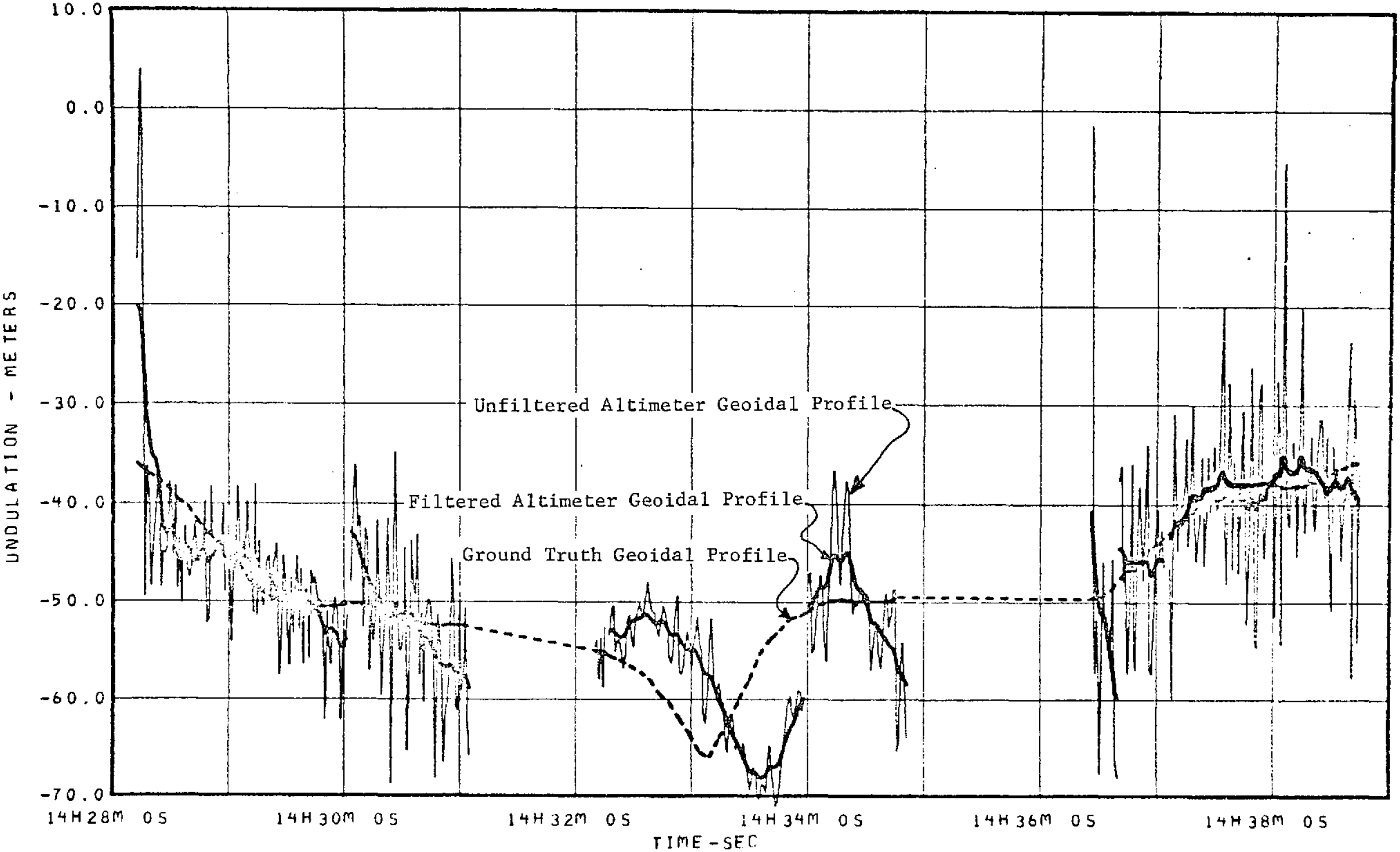

FIGURE 2. GEOID UNDULATIONS COMPUTED FROM SKYLAB PASS-7 ALTIMETRY DATA
ALTIMETER PRECISION = 2 m.

RECOMMENDATIONS

We recommend that additional efforts be considered for (1) determining the effect of sea state bias on the altimetry geoid determination, and (2) geodetic reduction and analysis of all other altimetry passes from missions SL-2 and SL-3. A copy describing, in detail, this additional effort has been sent to the technical monitor.

NEXT PERIOD

The investigation plans for the next period include

(1) Continuation of the efforts for the application of the collocation technique to filter the remaining altimeter passes (4, 6, 9).

(2) Continuation of the studies into the correlation between altimetry geoid and the geophysical data (bathymetric and gravimetric) along the subsatellite profiles.

(3) Initiate the preparation of the Final Report.

TRAVEL PLANS

No plans for travel during the next period are anticipated at this time.

APPENDIX A

FILTERING OF THE SATELLITE ALTIMETRY DATA

The error sources that affect the satellite altimeter measurements can be broadly grouped into three types

(1) Those that affect the accuracy of the measurement itself

(2) Those that cause the measured surface (instantaneous mean sea level) to be different from the geoid

(3) Those that contribute to the uncertainity in the geocentric position of the satellite.

Since the altimetry height which is the height of the satellite above the (instantaneous) mean sea level is linearly related to the geoid undulation at the satellite sub-point (Gopalapillai, 1974), the undulations, contaminated with both the systematic and random errors, can be assumed to be the observations instead of the ocean-satellite distances.

Let these observations, x, be modeled as follows (Moritz, 1972)

$$x = AX + s + n \tag{A-1}$$

where AX is a set of linear functions representing the systematic part of x with A and X being the design matrix and the unknowns respectively. s, a vector of systematic quantities which are random in nature, is called the "signal" and n, "the noise", is the vector of the measuring errors. This model is well illustrated in Figure A-1.

We have to determine the curve shown on top (full line) by means of discrete observations (small circles), which are furthermore affected by observational errors n. These observations have to be filtered for the systematic parts AX and s, both of which are of importance. For example, in the case of the altimetry data, AX will represent the systematic errors identified and modeled, s will represent the geoid undulations, and n the observational errors.

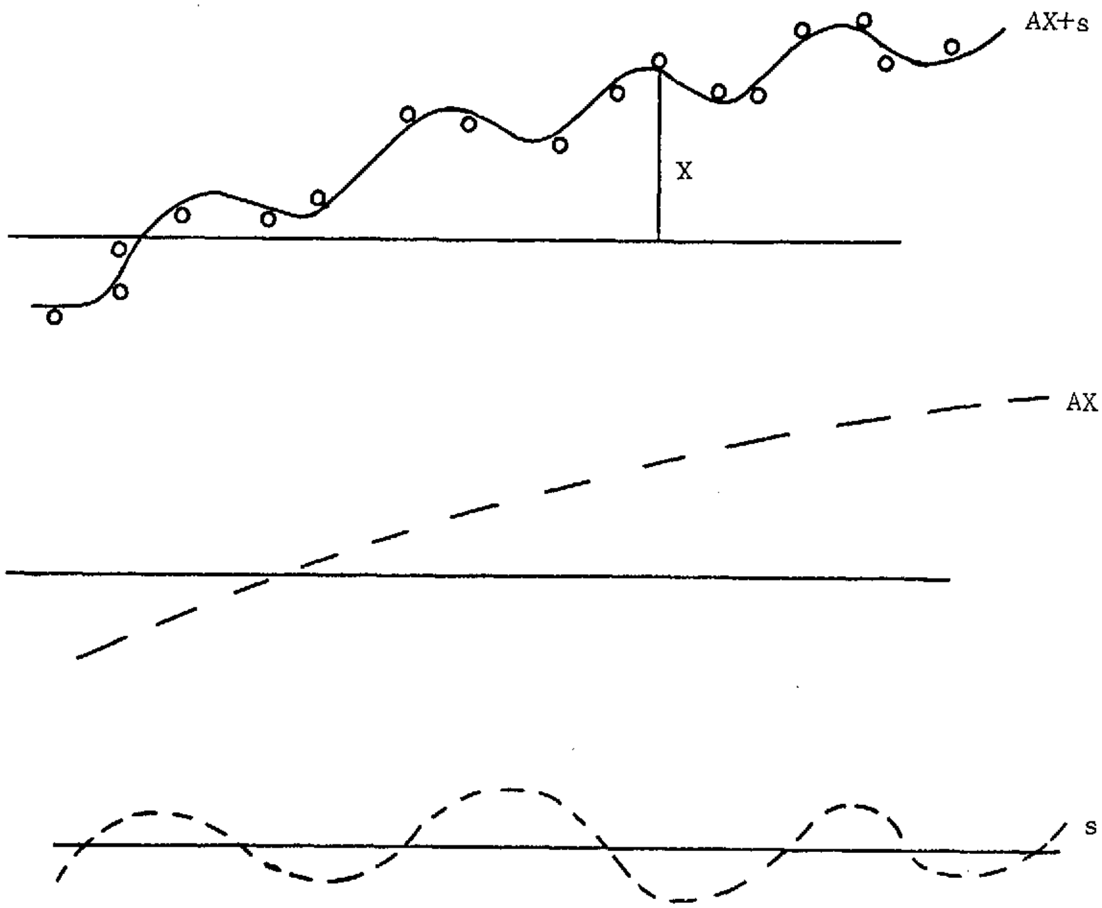

FIGURE A-1. ILLUSTRATION OF A LEAST SQUARE COLLOCATION MODEL

If we consider the signal to be the short periodic deviations of the altimetry geoid from the ground truth geoid then AX will represent the combined effect of the systematic errors and the ground truth geoid undulations.

Determination of the parameters X is called estimation; computation of s at points other than the observation points is prediction and the removal of the noise, n, is filtering. Consequently, Least Squares Collocation is a combination of some or all of the above processes. In the following discussions, the application of this Collocation technique to the satellite altimetry data processing will be generalized to all three processes so that the formulas for one or two specific processes can be deduced from the general ones.

Let us assume that we wish to predict the signal, s; at an arbitary number of "computation points" which may be different from the "data" points. Denote the number of such computation points by p and that of the data points by q. Define a vector v given by

$$v = [s_1^1 \; s_2^1 \; \ldots\ldots \; s_p^1 \; z_1 \; z_2 \; \ldots \; z_q]^T = [s^{1T} \; z^T]^T \tag{A-2}$$

where,

$$z = s^1 + n \tag{A-3}$$

and T denoting the transpose. Consequently, v is a vector of p + q random variables that enter into the problem.

The covariance matrix Q of this vector v may be written as a partitioned matrix:

$$Q = \begin{bmatrix} C_{s^1 s^1} & C_{s^1 z} \\ C_{z s^1} & C_{zz} \end{bmatrix} \tag{A-4}$$

Here,

$$C_{s^1 s^1} = \mathrm{Cov}\,(s^1, s^1)$$

denotes the covariance matrix of the signal s^1. Similarly, C_{zz} is the covariance matrix of the random vector z and C_{s^1z} and C_{zs^1} are the cross covariances between these quantities. Moritz (1972) has shown that

$$C_{zz} = C_{xx} = C_{ss} + C_{nn}$$
$$C_{zs^1} = C_{xs^1} = C_{ss^1} \tag{A-5}$$
$$C_{s^1z} = C_{s^1x} = C_{s^1s}.$$

Then, the matrix Q can be rewritten in the form

$$Q = \begin{bmatrix} C_{s^1s^1} & C_{s^1s} \\ C_{ss^1} & C_{ss} + C_{nn} \end{bmatrix} \tag{A-6}$$

Introducing the vector, v, as given by equation (A-2), equation (A-1) can be rewritten in the following form:

$$AX + BV - x = 0 \tag{A-7}$$

where

$$B = \begin{bmatrix} 0 & I \end{bmatrix} \tag{A-8}$$

which is a q by (q + p) matrix. Minimizing the squares v^1pv where,

$$P = Q^{-1} \tag{A-9}$$

with the side equation given by (A-7), the solution to our basic problems (estimation, filtering and prediction) are given by (Moritz, 1972)

$$X = (A^T\bar{C}^{-1} A)^{-1} A^T\bar{C}^{-1}x \tag{A-10}$$

$$v = QB^T\bar{C}^{-1}(x-AX) \tag{A-11}$$

and $$s^1 = C_{s^1s}\bar{C}^{-1}(x-AX) \tag{A-12}$$

where,

$$\bar{C} = C_{ss} + C_{nn} \quad . \tag{A-13}$$

If the computation points for s and the data points are identical, then,

$$s^1 = s \tag{A-14}$$

and, from (A-12)

$$s = C_{ss}\bar{C}^{-1}(x - AX) \tag{A-15}$$

Consequently, the filtered observation x is given by

$$x = AX + s \tag{A-16}$$

In order to evaluate these equations, all the matrices are known except the covariance matrices C_{ss} and C_{s^1s}.

These covariances describe the behavior of the signal in the coordinate frame in which the observations are made. For example, in the case of the satellite altimetry data, the frame is time which is in turn correlated with the latitude and longitude of points on the surface of the earth. If the signal is a set of geoid undulations, several empirical and numerical covariance functions are available (e.g., Tscherning and Rapp, 1974). On the other hand, if the signal is a set of differences in undulations between the altimetry and the ground truth geoids, empirical or numerical covariance functions may be computed as described in (Moritz, 1972) using some sample altimetry and ground truth data in the area under investigations.

Some further comments about the evaluation of equations (A-10) to (A-12) may be appropriate at this point. There are two matrices $-(A^T\bar{C}^{-1}A)$ and $\bar{C}$- which need to be inverted. The size of the first matrix is the number (m) of unknowns in the X vector and that of the second matrix is q. For the problem to be over-determined, q must be greater than m. Consequently, if q is very large, the evaluation of these equations may become uneconomical and difficult if not impossible. Therefore, modification of these equations which would avoid the inversion of large matrices needs to be investigated.

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